Some Generalizations Of g^{**}-Open Sets in Topological Spaces

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Abstract—In this paper, we generated a new type of open sets, namely g^{**}-open sets in topological spaces, used to constructed new types of separation axioms g^{**}-T_i spaces (i = 0, ½, 1, 2) and characterized g^{**}-T_i spaces using g^{**}-open and g^{**}-closed sets. Further we defined new generalized closed sets, namely g^{**}g-closed and gg^{**}-closed sets and investigated some of their basic properties.

Keywords-*g^{**}-open (closed), g^{**}g-open (closed), gg^{**}-open (closed), g^{**}-T_i spaces (i = 0, ½, 1, 2.)*

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I. Introduction

The concept of generalized closed (g-closed) sets in a topological space was introduced by Levine [17] and concept of $T_{1/2}$ spaces and defined a new closure operator cl^* by using generalized closed sets. Levine [18] introduced the concept of semi open sets and semi continuity in a topological space. Bhattacharya et.al [6] introduced a new class of semi generalized open sets by means of semi open sets introduced by Levine [17]. Balachandran, et.al [5], introduced the concept of generalized continuous maps and generalized homeomorphism in a topological space. Sundaram et.al [24] defined the concept of semi generalized continuous maps and semi $T_{1/2}$ spaces. Pushpalatha et.al [21] introduced the concept of g^*s-closed sets and g^*s-continuous maps in a topological space. Sai Sundara Krishnnan et.al [22] introduced the concept of g^{**}-closed sets and defined the new class of homeomorphism in a topological space. Saravanakumar et.al [23] defined the concept of αg^{**}-closed sets, αg^{**}-continuous and αg^{**}-irresolute mappings and studied some their important properties. We begin with some basic concepts. A subset A of a topological space (X, τ) is called α-open [12] (resp. semi open [18]) if $A \subseteq int(cl(int(A)))$ (resp. $A \subseteq cl(int(A))$). Also A is said to be α-closed (resp. semi-closed) if X – A is α-open (resp. semi-open). A subset A of a topological space (X, τ) is said to be g-closed [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a open set in X. Its complement is called g-open. The collection of all α-open [12] (resp. semi-open [18], g-open [17]) subsets in (X, τ) is denoted by τ^α (resp. SO(X), GO(X)). The α-closure (resp. semi-closure, g-closure) of a subset A is smallest α-closed (resp. semi-closed, g-closed) set containing A and this is denoted by $\alpha cl(A)$ (resp. scl(A), gcl(A)). A subset A of a topological space (X, τ) is called g^{**}-open [22] if there exists an open set U such that $U \subseteq A \subseteq gcl(U)$. Its complement is called g^{**}-closed. The collection of all g^{**}-open sets is denoted by $G^{**}O(X)$. The $g^{**}cl(A)$ [22] is defined as the smallest g^{**}-closed set containing A. A subset A of a topological space (X, τ) is called sg^{**}-closed [7] (resp αg^{**}-closed [23]) if $scl(A) \subseteq U$ (resp. $\alpha cl(A) \subseteq U$) whenever $A \subseteq U$ and U is a g^{**}-open set in (X, τ).

In this paper we introduce the concept of new types of separation axioms g^{**}-T_i spaces (i = 0, ½, 1, 2) and characterized g^{**}-T_i spaces using through the operator g^{**}cl. and analysed g^{**}-T_i spaces using g^{**}-open and g^{**}-closed sets. Further we obtained the relationships between g^{**}-T_i spaces and studied some their basic properties. In addition, we generated g^{**}-closed sets and obtained new generalized closed set, namely generalized-g^{**}-closed (briefly gg^{**}-closed) sets and their important properties. Moreover, we obtained the relationships between generalized closed sets such as closed, g^{**}-closed, g^{**}g-closed, gg^{**}-closed sets. Throughout this paper, we denoted cl^{**} (or cl^*) by g-cl and we represented the topological space (X, τ) as X. Unless otherwise no separation axiom mentioned.

II. G^{**}-Separation Axioms

Definition 2.1. A topological space X is called a g^{**}-T_0 space if for each pair of distinct points $x, y \in X$, there exists a g^{**}-open set U such that either $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$.

Definition 2.2. A topological space X is called a g^{**}-T_1 space if for each pair of distinct points $x, y \in X$, there exists a g^{**}-open sets U and V contain x and y respectively such that $y \notin U$ and $x \notin V$.

Definition 2.3. A topological space X is called a g^{**}-T_2 space if for each pair of distinct points $x, y \in X$, there exists g^{**}-open sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \varnothing$.

Definition 2.4. Let X be a topological space and A be a subset of X. Then A is called a g^{**}-generalized closed (briefly g^{**}g-closed) set if $g^{**}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a g^{**}-open set in X.

Example 2.1. Let X = {a, b, c} and τ = {∅, X, {a}, {b}, {a, b}}. Then the collection of g^{**}g-closed sets is {∅, X, {a}, {b}, {c}, {a, c}, {b, c}}.

Remark 2.1. Union of two g^{**}g-closed sets need not be g^{**}g-closed.

From Example 2.1, Take A = {a} and B = {b}. Then A and B are g^{**}g-closed sets but $A \cup B$ = {a, b} is not g^{**}g-closed.

Remark 2.2. Intersection of two g^{**}g-closed sets need not be g^{**}g-closed.

If X = {a, b, c, d} and τ = {∅, X, {a}, {b}, {a, b}, {b, c, d}}, then the collection of g^{**}g-closed sets is {∅, X, {a}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Take A = {b, c} and B = {b, d}, but $A \cap B$ = {b} is not g^{**}g-closed.

Theorem 2.1. Let X be a topological space. If A is a g^{**}-closed set in X, then A is g^{**}g-closed.

Proof. Let A be a g^{**}-closed set in X and let U be a g^{**}-open set in X such that $A \subseteq U$. Then by Remark 3.18[22], we have that $A = g^{**}cl(A)$. This implies that $g^{**}cl(A) \subseteq U$ and by Definition 2.4, A is $g^{**}g$-closed.

Remark 2.3. Any $g^{**}g$-closed set need not be g^{**}-closed.

From Remark 2.2, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, d}, {a, c, d} are all $g^{**}g$-closed sets but not g^{**}-closed.

Definition 2.5. A topological space X is called a g^{**}-$T_{1/2}$ space if each $g^{**}g$-closed set of X is g^{**}-closed.

Theorem 2.2. Let X be a topological space. Then for a point $x \in X$, $x \in g^{**}cl(A)$ if and only if $V \cap A \neq \varnothing$ for any $V \in G^{**}O(X)$ such that $x \in V$.

Proof. Let F_0 be the set of all $y \in X$ such that $V \cap A \neq \varnothing$ for any $V \in G^{**}O(X)$ and $y \in V$. Now, we prove that $g^{**}cl(A) = F_0$. Let us assume $x \in g^{**}cl(A)$ and $x \notin F_0$. Then there exists a g^{**}-open set U of x such that $U \cap A = \varnothing$. This implies that $A \subseteq X - U$. Therefore $g^{**}cl(A) \subseteq X - U$. Hence $x \notin g^{**}cl(A)$. This is a contradiction. Hence $g^{**}cl(A) \subseteq F_0$. Conversely, let F be a set such that $A \subseteq F$ and $X - F \in G^{**}O(X)$. Let $x \notin F$. Then we have that $x \in X - F$ and $(X - F) \cap A = \varnothing$. This implies that $x \notin F_0$. Therefore $F_0 \subseteq F$. Hence $F_0 \subseteq g^{**}cl(A)$.

Theorem 2.3. Let X be a topological space and A be a subset of X. Then A is $g^{**}g$-closed if and only if $g^{**}cl(\{x\}) \cap A \neq \varnothing$ holds for every $x \in g^{**}cl(A)$.

Proof. Let U be any g^{**}-open set in X such that $A \subseteq U$. Let $x \in g^{**}cl(A)$. By assumption there exists a point $z \in g^{**}cl(\{x\})$ and $z \in A \subseteq U$. Therefore from Theorem 5.1, we have that $U \cap \{x\} \neq \varnothing$. This implies that $x \in U$. Hence A is a $g^{**}g$-closed set in X. Conversely, suppose there exists a point $x \in g^{**}cl(A)$ such that $g^{**}cl(\{x\}) \cap A = \varnothing$. Since $g^{**}cl(\{x\})$ is a g^{**}-closed set implies that $X - g^{**}cl(\{x\})$ is a g^{**}-open set. Since $A \subseteq X - g^{**}cl(\{x\})$ and A is $g^{**}g$-closed set, implies that $g^{**}cl(A) \subseteq X - g^{**}cl(\{x\})$. Hence $x \notin g^{**}cl(A)$. This is a contradiction.

Theorem 2.4. Let X be a topological space and A be the $g^{**}g$-closed set in X. Then $g^{**}cl(A) - A$ does not contain a non empty g^{**}-closed set.

Proof. Suppose there exists a non empty g^{**}-closed set F such that $F \subseteq g^{**}cl(A) - A$. Let $x \in F$. Then $x \in g^{**}cl(A)$, implies that $F \cap A = g^{**}cl(A) \cap A \supseteq g^{**}cl(\{x\}) \cap A \neq \varnothing$ and hence $F \cap A \neq \varnothing$. This is a contradiction.

Theorem 2.5. Let X be a topological space. Then for each $x \in X$, {x} is g^{**}-closed or $X - \{x\}$ is $g^{**}g$-closed.

Proof. Suppose that {x} is not g^{**}-closed. Then $X - \{x\}$ is not g^{**}-open. This implies that X is the only g^{**}-open set containing $X - \{x\}$ and hence $X - \{x\}$ is $g^{**}g$-closed.

Theorem 2.6. A topological space X is a g^{**}-$T_{1/2}$ space if and only if for each $x \in X$, {x} is g^{**}-open or g^{**}-closed.

Proof. Suppose that {x} is not g^{**}-closed. Then it follows from the assumption and Theorem 2.5, {x} is g^{**}-open. Conversely, let F be a $g^{**}g$-closed set in X. Let $x \in g^{**}cl(F)$. Then by the assumption {x} is either g^{**}-open or g^{**}-closed.

Case (i): Suppose that {x} is g^{**}-open. Then by Theorem 2.2, $\{x\} \cap F \neq \varnothing$. This implies that $g^{**}cl(F) = F$. Therefore X is a g^{**}-$T_{1/2}$ space.

Case (ii): Suppose that {x} is g^{**}-closed. Let us assume $x \notin F$. Then $x \in g^{**}cl(F) - F$. This is a contradiction. Hence $x \in F$. Therefore X is a g^{**}-$T_{1/2}$ space.

Theorem 2.7. A space X is g^{**}-T_1 if and only if for any $x \in X$, {x} is g^{**}-closed.

Proof. Follows from Definitions 3.16[22] and 2.2.

Remark 2.4. (i) Every g^{**}-$T_{1/2}$ space is g^{**}-T_0, but converse need not be true.

Let X = {a, b, c, d} and $\tau = \{\varnothing, X, \{a\}, \{a, b\}, \{a, b, d\}\}$. Then X is a g^{**}-T_0 space. Also {a, c, d} is a $g^{**}g$-closed set but not g^{**}-closed. Hence X is not g^{**}-$T_{1/2}$.

(ii) Every g^{**}-T_1 space is g^{**}-$T_{1/2}$, but converse need not be true.

Let X = {a, b, c} and $\tau = \{\varnothing, X, \{b\}, \{a, b\}, \{b, c\}\}$. Then X is a g^{**}-$T_{1/2}$ space but not g^{**}-T_1.

(iii) Every g^{**}-T_2 space is g^{**}-T_1, but converse need not be true.

Let X = {a, b, c} and $\tau = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$. Then X is a g^{**}-T_1 space but not g^{**}-T_2.

Remark 2.5. From Definitions 2.1, 2.2, 2.3, 2.5, Theorems 2.5, 2.6, 2.7, Remark 3.4[22], 2.4, we have the following relationship diagram

$$g^{**}\text{-}T_2 \;\underset{\not\leftarrow}{\rightarrow}\; g^{**}\text{-}T_1 \;\underset{\not\leftarrow}{\rightarrow}\; g^{**}\text{-}T_{1/2} \;\underset{\not\leftarrow}{\rightarrow}\; g^{**}\text{-}T_0$$

where $A \rightarrow B$ represent A implies B, $A \not\rightarrow B$ represent A does not imply B.

Definition 2.6. Let A be subset of a topological space X. Then g^{**}-interior of A is defined as union of all g^{**}-open sets contained in A.

Thus $g^{**}int(A) = \cup \{U : U \in G^{**}O(X) \text{ and } U \subseteq A\}$.

Theorem 2.8. Let $\{A_\alpha : \alpha \in J\}$ be the collection of g^{**}-open sets in a topological space X. Then $\cup_{\alpha\in J} A_\alpha$ is also a g^{**}-open set in X.

Proof. Since A_α is g^{**}-open, then by Theorem 3.3[22], we have that $A_\alpha \subseteq g\text{-}cl(int(A_\alpha))$. This implies that $\cup_{\alpha\in J} A_\alpha \subseteq \cup_{\alpha\in J} (g\text{-}cl(int(A_\alpha))) \subseteq g\text{-}cl(int(\cup_{\alpha\in J} A_\alpha))$. Hence $\cup_{\alpha\in J} A_\alpha$ is a g^{**}-open set in X.

Theorem 2.9. Let X be a topological space. A subset A of X is g^{**}-closed in X if and only if $g\text{-}int(cl(A)) \subseteq A$.

Proof. If A is a g^{**}-closed in X, then $X - A$ is g^{**}-open. By Theorem 3.3[22], we have that $X - A \subseteq g\text{-}cl(int(X - A))$. This implies that $X - A \subseteq g\text{-}cl(int(X - A)) = g\text{-}cl(X - cl(A)) =$

X – g-int(cl(A)) and hence g-int(cl(A)) $\subseteq$ A. Conversely, let g-int(cl(A)) $\subseteq$ A. Then X – A $\subseteq$ X – g-int(cl(A)) = g-cl(X – cl(A)) = g-cl(int(X – A)) and hence X – A $\subseteq$ g-cl(int(X – A)). Then by Theorem 3.3[22], we have that X – A is g^{**}-open. Therefore A is g^{**}-closed.

Theorem 2.10. Let $\{A_\alpha : \alpha \in J\}$ be the collection of g^{**}-closed sets in a topological space X. Then $\cap_{\alpha\in J} A_\alpha$ is also a g^{**}-closed set in X.

Proof. Follows from Theorem 2.8 and 2.9.

Theorem 2.11. Let A be a subset of a topological space X. Then
(i) g^{**}int(A) is a g^{**}-open set contained in A;
(ii) g^{**}cl(A) is a g^{**}-closed set containing A;
(iii) A is g^{**}-closed if and only if g^{**}cl(A) = A;
(iv) A is g^{**}-open if and only if g^{**}int(A) = A;
(v) g^{**}int(A) = X – g^{**}cl(X – A);
(vi) g^{**}cl(A) = X – g^{**}int(X – A).

Proof. Follows from Definitions 3.17[22], 2.6, Theorem 2, 8, 2.9 and 2.10.

Theorem 2.12. Let X be a topological space. If A and B are two subsets of X, then the following are hold:
(i) If A $\subseteq$ B, then g^{**}int(A) $\subseteq$ g^{**}int(B);
(ii) g^{**}int(A $\cup$ B) = g^{**}int(A) $\cup$ g^{**}int(B);
(iii) g^{**}int(A $\cap$ B) $\subseteq$ g^{**}int(A) $\cap$ g^{**}int(B).

Proof. Follows from Definition 2.6, Theorem 2.8 and Remark 3.16[22].

Definition 2.7. A subset A of a topological space X is said to be a g^{**}-generalized open (briefly g^{**}g-open) set if X – A is a g^{**}g-closed set in X.

Example 2.2. Let X = {a, b, c} and τ = {∅, X, {a}, {b}, {a, b}}. Then the collection of g^{**}g-open sets is {∅, X, {a}, {b}, {a, b}, {a, c}, {b, c}}.

Theorem 2.13. A subset A of a topological space X is g^{**}g-open if and only if V $\subseteq$ g^{**}int(A) whenever V $\subseteq$ A and V is g^{**}-closed in X.

Proof. Let A be a g^{**}g-open set in X and let V be a g^{**}-closed in X such that V $\subseteq$ A. Then X – A $\subseteq$ X – V and X – V is g^{**}-open. Since A is g^{**}g-open, X – A is g^{**}g-closed. This implies that g^{**}cl(X – A) $\subseteq$ X – V. Also by Theorem 2.11(vi), we have that X – g^{**}int(A) = g^{**}cl(X – A) $\subseteq$ X – V. Therefore V $\subseteq$ g^{**}int(A). Conversely, let A = X – B and let U be g^{**}-open such that B $\subseteq$ U. This implies that X – U $\subseteq$ X – B and X – U is g^{**}-closed. Then by hypothesis, X – U $\subseteq$ g^{**}int(X – B). Also by Theorem 2.11(v), X – U $\subseteq$ X – g^{**}cl(B). Therefore g^{**}cl(B) $\subseteq$ U. Thus B is g^{**}g-closed and hence X – B = A is g^{**}g-open.

Definition 2.8. Let X be a topological space. Then a subset A of X is said to be generalized-g^{**}-open (briefly gg^{**}-open) if F $\subseteq$ g^{**}int(A) whenever F $\subseteq$ A and if F is closed in X. A subset A of X is said to be generalized-g^{**}-closed (briefly gg^{**}-closed) if X – A is gg^{**}-open.

Example 2.3. Let X = {a, b, c, d} and τ = {∅, X, {b}, {b, c}, {a, b, c}, {b, c, d}}. Then (i) the collection of gg^{**}-open sets is {∅, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}.

(ii) Also the collection of gg^{**}-closed sets is {∅, X, {a}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}.

Theorem 2.14. A subset A of a topological space X is gg^{**}-closed if and only if g^{**}cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Proof. Let A be a gg^{**}-closed set in X and let U be open in X such that A $\subseteq$ U. Then X – U $\subseteq$ X – A and X – U is closed. Since A is gg^{**}-closed, X – A is gg^{**}-open. This implies that X – U $\subseteq$ g^{**}int(X – A). Also by Theorem 2.11(v), we have that X – U $\subseteq$ X – g^{**}cl(A). Therefore g^{**}cl(A) $\subseteq$ U. Conversely, let A = X – B and let F be closed such that F $\subseteq$ B. This implies that X – B $\subseteq$ X – F and X – F is open. Then by hypothesis, g^{**}cl(X – B) $\subseteq$ X – F. Also by Theorem 2.11(vi), X – g^{**}int(B) = g^{**}cl(X – B) $\subseteq$ X – F. Therefore F $\subseteq$ g^{**}int(B). Thus B is gg^{**}-open and hence X – B = A is gg^{**}-closed.

Remark 2.6. Union of two gg^{**}-closed sets need not be gg^{**}-closed.

From Example 2.1, Take A = {a} and B = {b}. Then A and B are gg^{**}-closed sets but A $\cup$ B = {a, b} is not gg^{**}-closed.

Remark 2.7. Intersection of two gg^{**}-closed sets need not be gg^{**}-closed.

If X = {a, b, c, d} and τ = {∅, X, {a}, {b}, {a, b}, {b, c, d}}, then the collection of gg^{**}-closed sets is {∅, X, {a}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Take A = {b, c} and B = {b, d}, but A $\cap$ B = {b} is not gg^{**}-closed.

Theorem 2.15. Let A be a subset of a topological space X.
(i) If A is a closed set of X, then A is g^{**}-closed in X;
(ii) If A is a g^{**}-closed set of X, then A is g^{**}g-closed in X;
(iii) If A is a g^{**}g-closed set of X, then A is gg^{**}-closed in X.

Proof. (i) Let A be a closed set in X. Then cl(A) = A. Since g-int(cl(A)) $\subseteq$ cl(A), implies that g-int(cl(A)) $\subseteq$ cl(A) = A and hence g-int(cl(A)) $\subseteq$ A. Then by Theorem 2.9, we have that A is g^{**}-closed.

(ii) Let A be a g^{**}-closed set in X and U be a g^{**}-open set in X such that A $\subseteq$ U. Since A is g^{**}-closed and by Theorem 2.11(iii), g^{**}cl(A) = A $\subseteq$ U. Therefore g^{**}cl(A) $\subseteq$ U and hence A is g^{**}g-closed.

(iii) Let A be a g^{**}g-closed set in X and U be a open set in X such that A $\subseteq$ U. Then by Remark 3.4[22], we have that U is g^{**}-open. Since A is g^{**}g-closed and by Definition 2.4, g^{**}cl(A) $\subseteq$ U and hence A is gg^{**}-closed.

Remark 2.8. Every g^{**}g-closed set of X is gg^{**}-closed, but converse need not be true.

From Theorem 2.15, we have that every $g^{**}g$-closed set of X is gg^{**}-closed.

Let X = {a, b, c, d} and τ = {∅, X, {b}, {b, c}, {a, b, c}, {b, c,d}}. Then (i) the collection of $g^{**}g$-closed sets is {∅, X, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}}.

(ii) Also the collection of gg^{**}-closed sets is {∅, X, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, b, d}, {a, c, d}}.

Thus {a, b, d} is a gg^{**}-closed set but not $g^{**}g$-closed.

Theorem 2.16. If A is both open and gg^{**}-closed set in X, then A is g^{**}-closed.

Proof. Since A is open and gg^{**}-closed, $g^{**}cl(A) \subseteq A$ and hence $g^{**}cl(A) = A$. This implies that A is g^{**}-closed.

Theorem 2.17. If A is a gg^{**}-closed subset of X, then $g^{**}cl(A) - A$ does not contain any nonempty closed set.

Proof. Let F be a closed subset of $g^{**}cl(A) - A$. Then $A \subseteq X - F$. Since A is gg^{**}-closed and X – F is open, implies that $g^{**}cl(A) \subseteq X - F$. Therefore $F \subseteq (X - g^{**}cl(A)) \cap g^{**}cl(A) = \emptyset$.

Theorem 2.18. Let $x \in X$. Then{x} is closed or X – {x} is gg^{**}-closed in X.

Proof. Suppose {x} is closed nothing to prove. Suppose {x} is not closed. Then X – {x} is not a open set. Therefore X is the only open set containing X – {x}. Hence $g^{**}cl(X - \{x\}) \subseteq X$. This implies that X – {x} is gg^{**}-closed.

Theorem 2.19. Let X be a topological space. Then the following conditions are equivalent:
(i) every gg^{**}-closed set of X is g^{**}-closed;
(ii) for each $x \in X$, singleton {x} is closed or g^{**}-open in X;
(iii) for each $x \in X$, singleton {x} is closed or open in X;
(iv) X is a $T_{1/2}$ space.

Proof. (i) ⇒ (ii). Let $x \in X$. Suppose {x} is closed nothing to prove. Suppose {x} is not closed. By Theorem 2.18, X – {x} is a gg^{**}-closed set. Therefore by assumption X – {x} is g^{**}-closed. Hence {x} is g^{**}-open.

(ii) ⇒ (iii) Suppose {x} is closed nothing to prove. Suppose {x} is not closed. Then {x} is g^{**}-open, implies that $\{x\} \subseteq$ g-cl(int{x}). Obvious int{x} = {x} otherwise {x} is not g^{**}-open and hence {x} is open.

(iii) ⇒ (iv). Obviously.

(iv) ⇒ (i). Let A be a gg^{**}-closed set. Now to prove that A is a g^{**}-closed set in X, that is to prove that $g^{**}cl(A) \subseteq A$. Let $x \in g^{**}cl(A)$. By assumption {x} is open or closed.

Case (i): Suppose that {x} is open. Then {x} is g^{**}-open. By using Theorem 2.2, $\{x\} \cap A \neq \emptyset$, this implies that $x \in A$.

Case (ii): Suppose that {x} is closed. It follows from Theorem 2.17 that $g^{**}cl(A) - A$ does not contain {x}. This implies that $x \in A$.

Hence $g^{**}cl(A) \subseteq A$. Therefore A is g^{**}-closed in X.

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